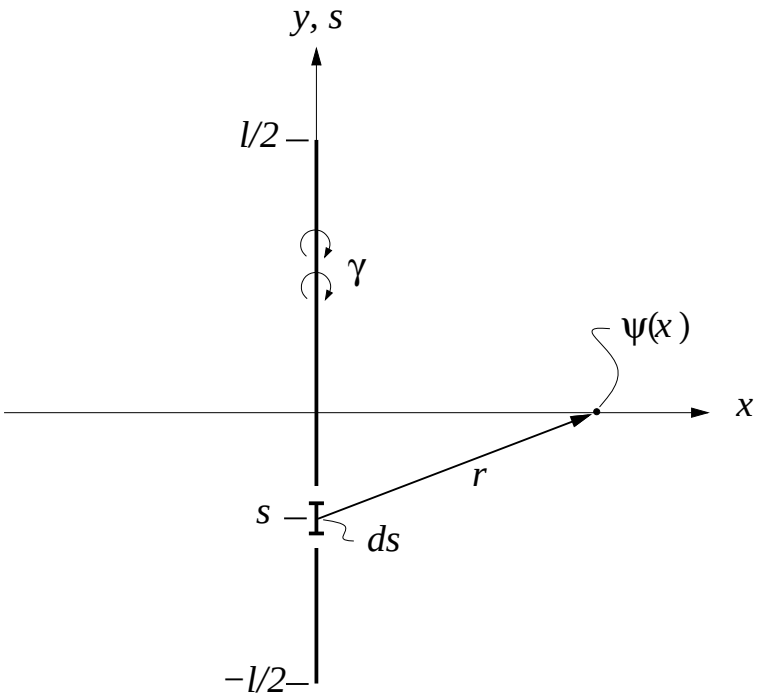
A vertical vortex sheet of constant strength γ is centered on the origin as shown. A small piece ds of the sheet is therefore a point vortex of circulation $d\Gamma = \gamma\, ds$. The stream function contribution of this piece at some location x on the axis is therefore given by

$$d\psi \;=\; \frac{d\Gamma}{2\pi}\,\ln r \;=\; \frac{\gamma\, ds}{2\pi}\,\ln r$$

a) Determine the overall stream function $\psi(x)$ on the axis, due to the entire sheet. (This is a subset of determining $\psi(x, y)$ on the entire plane).

b) Determine the vertical velocity $v(x)$ of the sheet on the x-axis. Also determine the vertical velocity $v_{\text{point}}(x)$ of a point vortex placed on the origin, with the same total vortex strength Γ as the sheet. Explicitly state the relation between γ and Γ in this case.

c) Plot $v(x)$ and $v_{\text{point}}(x)$ overlaid. Set the plot scale to ignore the singularity in v_{point} at $x = 0$. Describe how the two curves relate at large x distances.



From CRC Handbook:

$$\int \ln(x^2 + a^2)\, dx \;=\; x\,\ln(x^2 + a^2) \;-\; 2x \;+\; 2a\,\arctan(x/a) \;+\; C$$